AN INVESTIGATION OF THE PHASE TRANSITION IN THE EXTENDED BAG MODEL

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Abstract

By proposing an Ansatz for the pressure (measured in terms of the bag constant) of the hadronic gas in equilibrium, we have formulated a rather simple phenomenological model (the extended bag model) which allows one to analytically investigate the bulk thermodynamic propertities in the vicinity of the phase transition. This makes it also possible to take into account the nonperturbative vacuum effects from both sides of the equilibrium condition. As an example of our approach, we have calculated the crossover (critical or transition) temperature T_c and the critical chemical potential μ_c (as functions of the bag constant) from the noninteracting quark-gluon plasma state equation. Our results for $T_c(N_f = 0) = 241.5\ MeV$ and $T_c(N_f = 2) = 160.6\ MeV$ are in good agreement with recent lattice data. The extensive densities such as the entropy density, specific heat, etc are calculated as well. A general scheme how to calculate the latent heat, the critical energy density, etc within the extended bag model is also described and it is applied to the two models of the hadronic gas phase.

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I. The quark-gluon plasma (QGP) phase is a necessary step in the evolution of the Exited Matter from Big Bang to the present days. Apparently, the only way to study this phase of the expansion of the Universe is nuclear (heavy ion) collisions at high energies which makes it possible "to recreate conditions akin to the first moments of the Early Universe, the Big Bang, in the laboratory" [1]. Because of the confinement phenomenon, the nonperturbative vacuum structure must play a very important role in the transition from QGP to the formation of the hadronic particles (i. e., hadronization) and vice versa. As it was underlined in our papers [2], any correct model of the nonperturbative effects such as quark confinement or dynamical chiral symmetry breaking (DCSB) becomes a model of the true QCD ground state (i. e., the nonperturbative vacuum) and the other way around. Thus the difference between the perturbative (always normalizable to zero) and the nonperturbative vacua appears to be necessarily nonzero and finite to describe the above mentioned nonperturbative phenomena at zero temperature. The existence of the finite vacuum energy per unit volume - the bag constant [3, 4] - becomes important for a realistic calculation of the transition data from the hadronic gas (HG) to QGP phases at nonzero temperature as well.

There are two main approaches to investigate QGP, namely the resummed finite-temperature perturbation theory (effective field theory method) [5, 6] and the lattice one [7]. The former breaks down after the fifth order in the QCD coupling constant g because of the severe infrared divergences in the Braaten-Pisarski-Kapusta (BPK) series in powers of $g^{m+2n} \ln^n g$ [5, 6, 8], but it smoothly incorporates the case of nonzero chemical potential(s). The latter is a powerful nonperturbative tool to calculate equations of state for both phases. However, up to now, there are no realistic lattice data available for nonzero chemical potential(s) (for problems to introduce it on the lattice see, for example, recent paper [9]).

As was emphasized in Ref. [10], at present the phase transition at finite baryon chemical potential(s) can be only studied within the phenomenological models. The most popular among them are, of course, the bag-type models [11], which differ from each other by modelling the equation of state of the hadronic phase [12, 13]. Within the framework of the

bag-type models, a phase transition between the HG and QGP phases is constracted via the Gibbs criteria for a phase equilibrium. So the phase transition is necessary of the first order. This means that the thermodynamic quantities of interest are discontinuous across the critical curve. Our main goal here is to propose a new bag-type model, the so-called extended bag model. It complements the standard bag model by an Ansatz for the pressure (measured in terms of the bag constant) which allows one to correctly take into account the nonperturbative vacuum effects from both sides of the equilibrium condition. We have calculated the critical chemical potential μ_c and, especially, the crossover (critical or transition) temperature T_c, at which deconfinement phase transition can occur and chiral symmetry is restored, in terms of the bag constant. Our numerical results for T_c are in good agreement with recent lattice data (see below). The extended bag model makes it also possible to analytically investigate the bulk thermodynamic propertities in the vicinity of the phase transition.

II. In the bag-type models the equation of state of the QGP phase is usually approximated by the thermal perturbation theory as the ideal (noninteracting) gas consisting of gluons and massless quarks. It determines the dependence of the QGP thermodynamical quantities such as energy density ϵ and pressure P on the thermodynamical variables, temperature T and quark chemical potentials μ_f. There exist excellent reviews on the physics of the QGP (see, for example, Refs. [14, 15]), as well as on the phase transitions in it [16]. The pressure (i. e., the thermodynamic potential Ω, apart from the sign) for the noninteracting QGP is given as follows [15]

$$P = \frac{1}{3} f_{SB} T^4 + \frac{N_f}{2} \mu_f^2 T^2 + \frac{N_f}{4\pi^2} \mu_f^4 - B, \tag{1}$$

where B is the bag constant (see below), while N_f is the number of different quark flavours. In what follows we will consider the values $N_f = 0, 1, 2$ since the inclusion of the strange (s) quark requires a special treatment [17]. The value $N_f = 0$ describes the case of the pure gluon plasma. Note also that the state equation (1) is derived by neglecting quark current masses. The constant f_{SB}, entering the equation of state (1),

$$f_{SB} \equiv f_{SB}(N_f) = \frac{\pi^2}{5}\Big(\frac{8}{3} + \frac{7}{4}N_f\Big) \tag{2}$$

is the Stefan-Boltzmann (SB) constant, which determines the ideal (noninteracting gluons and massless quarks) gas limit. Obviously, for $N_f = 0$ it equals to the standard SB constant of the ideal gluon gas. It is well known that the energy density ϵ of the noninteracting QGP can be obtained from the thermodynamic pothential (1) as follows

$$\epsilon = 3P + 4B, \tag{3}$$

i. e. the bag pressure B determines the deviation from the ideal gas relation. Let us make a few remarks in advance. Our calculations of T_c and μ_c for the noninteracting QGP are not based on the bag model state equation (3). The constraint, determining the phase transition, will be obtained with the help of an Ansatz which is beyond the standard bag model and it is general (see below, part III). We will use numerical values of the bag constant which was obtained from a completely different source, namely it was calculated in the zero modes enhancement (ZME) model of the true QCD vacuum on account of the instanton contributions as well. (see below, part IV).

III. The Gibbs conditions for the phase equilibrium between the HG and QGP phases at $T = T_c$ are formulated as follows [11]

$$P_h = P_q = P_c; \quad T_h = T_q = T_c; \quad 3\mu_f = \mu_c, \tag{4}$$

where subscripts h, q and c refer to HG, QGP phases and transition (critical or crossover) region, respectively. At the same time, the difference between $\epsilon_q - \epsilon_h$ at $T = T_c$ can remain finite (nonzero) and determines the latent heat (LH), ϵ_{LH}.

Let us formulate our primary assumption (Ansatz) now. The state equation for the hadronic phase (the left hand side of the equilibrium condition (4)) is strongly model dependent [11-13]. However, in any model the pressure P_h at any values of T and μ_h, in particular at $T = T_c$ and $\mu_h = \mu_c$, can be measured in terms of the above mentioned bag constant, i.e. let us put

$$P_h(T_c, \mu_c) = \frac{b_h}{a_h + N_f} B, \tag{5}$$

where the so-called parametric functions $b_h \equiv b_h(T_c, \mu_c)$ and $a_h \equiv a_h(T_c, \mu_c)$ describe the details of the HG phase at the phase boundary. Evidently, they may only depend on the set of independent dimensionless variables (by definition) which characterize the HG phase. For example,

$$\begin{aligned} a_h &\equiv a_h(T_c, \mu_c) = a_h(x_c, t_c, y_c, z_c), \\ b_h &\equiv b_h(T_c, \mu_c) = b_h(x_c, t_c, y_c, z_c), \end{aligned} \tag{6}$$

where

$$x_c = \frac{\mu_c}{T_c}, \quad t_c = \frac{\tilde{B}^{1/4}}{T_c}, \quad y_c = \frac{\mu_c}{m}, \quad z_c = \mu_c R_0, \tag{7}$$

and m denotes the hadron mass while R_0 denotes radius of the nucleon, so it allows one to take into account finite size effects due to hard core repulsion between nucleons (extended volume corrections) [18]. These variables are independent and all other possible dimensionless variables are obtained by combination of these, for example, $R_0 T_c = z_c/x_c, \quad m/T_c = x_c/y_c, \quad \mu_c/\tilde{B}^{1/4} = x_c/t_c, etc.$ Also the set of independent dimensionless variables at the phase boundary (7) may be extended in order to treat the HG phase in more sophisticated way. However, in any case the parametric functions should be symmetric, i. e. $a_h(T_c, \mu_c) = a_h(-T_c, -\mu_c)$ and $b_h(T_c, \mu_c) = b_h(-T_c, -\mu_c)$.

From Eqs. (1) and (4-5) at $T = T_c$ and $\mu_f = \mu_c/3$, one obtains

$$f_{SB}(N_f) T_c^4 + \frac{N_f}{6} \mu_c^2 T_c^2 + \frac{N_f}{108\pi^2} \mu_c^4 = \frac{3}{a_h + N_f} \tilde{B}, \tag{8}$$

where we introduced a new "physical" (effective) bag constant as follows

$$\tilde{B} = (b_h + a_h + N_f) B, \tag{9}$$

and it linearly depends on N_f as it should be at log-loop level (see below, part IV). In connection with our Ansatz (5) a few remarks are in order. Its alternative parametrization

with respect to N_f, namely $P_h(T_c, \mu_c) = (b_h^{'}/a_h^{'} N_f + 1)B$ leads to the effective bag constant as $\tilde{B} = (b_h^{'} + a_h^{'} N_f + 1)B$. The parameteric functions b_h and a_h, as well as $b_h^{'}$ and $a_h^{'}$, as was mentioned above, may, in principle, arbitrary depend on dimensionless variables (7). If, for example, $a_h^{'}$ vanishes at $\mu_c = 0$ then the linear dependence of $\tilde{B}$ on N_f will be spoiled. In other words, the choosen parametrization guarantees the linear dependence of $\tilde{B}$ on N_f and the alternative one does not.

From now on T_c and μ_c will be calculated in terms of $\tilde{B}$ and not that of old B, i. e. a definite numerical value will be assigned to $\tilde{B}$. So we consider $\tilde{B}$ as the physical bag constant, while B as unphysical "bare" one. The bag constant is a universal one and it represents the complex nonperturbative structure of the QCD true vacuum. Thus the proposed Ansatz, allows one to take into account nonperturbative vacuum effects (parametrized in terms of $\tilde{B}$) from both sides of the equilibrium condition (4). However, it still remains dependent on the arbitrary parameter a_h. In order to eliminate this dependence, let us normalize the thermodynamic potential at the phase boundary in Eq. (8) to the standard SB constant (2). This yields

$$\tilde{f}_{SB}(N_f) = (N_f + a_h) f_{SB}(N_f) = f_{SB}(0) \quad at \quad N_f = 0, \tag{10}$$

and one immediately arrives at $a_h \equiv a_h(T_c, \mu_c) = 1$. This is our normalization condition and it leads to good numerical results for T_c and μ_c (see below). So the general constraint (8) finally becomes uniquely determined, namely

$$f_{SB} T_c^4 + \frac{N_f}{6} \mu_c^2 T_c^2 + \frac{N_f}{108\pi^2} \mu_c^4 = \frac{3}{N_f + 1} \tilde{B}, \tag{11}$$

and consequently allows one to investigate the bulk thermodynamic quantites in the vicinity of a critical point. Let us emphasize the important observation that the numerical values of T_c and μ_c calculated from the constraint (11) do not depend on how one approximates the equation of state of the hadronic phase. In contrast to the standard bag-type models (differed from each other by modelling the hadronic phase), in the extended bag model their values depend only on $\tilde{B}$ which incorporates nonperturbative vacuum effects from both sides of the Gibbs equilibrium condition (4) as it has been already emphasized above.

The QGP energy density and pressure, however, remain undetermined with our Ansatz (5) at this stage. In terms of $\tilde{B}$ they become

$$P_h(T_c) = P_q(T_c) = \frac{b_h}{(N_f+1)[(N_f+1)+b_h]}\tilde{B} \tag{12}$$

and because of Eq. (3),

$$\epsilon_q(T_c) \equiv \epsilon_c = \frac{3b_h + 4(N_f+1)}{(N_f+1)[(N_f+1)+b_h]}\tilde{B}. \tag{13}$$

As mentioned above, the unknown b_h reflects the fact that the hadronic phase state equation is strongly model dependent. The unknown b_h is the price paied to determine the above mentioned physical quantities with our Ansatz. Precisely for this reason, the bag model state equation (3) plays no role in our numerical investigation of the phase transition with the constraint (11) from which T_c, as well as μ_c, can be derived. Below (see part VI) a general scheme, how to calculate b_h within our model, will be developed. Concluding this part, let us note that our model cetainly requires the coexistence regime between QGP and HG phases since $\epsilon_q(T_c)$ as given by Eq. (13) explicitly depends on b_h which describes details of the HG phase at $T = T_c$.

IV. Let us discuss the possible values of the bag constant itself now. The bag constant is the difference between the energy density of the perturbative and the nonperturbative QCD vacua (at zero temperature). The former one can always be normalized to zero, so the bag constant is defined as $\tilde{B} = -\epsilon$, where ϵ is the energy density of the nonperturbative vacuum. This is always negative, consequently the bag constant is always positive. Let us start from the so-called standard value. In the random instanton liquid model (RILM) [19] of the QCD vacuum, for a dilute ensemble, one has $\epsilon_I = -(1/4)(11 - \frac{2}{3}N_f) \times 1.0\ fm^{-4}$. Then the standard value of the bag constant is

$$\tilde{B}_{st} \equiv \tilde{B}_I = -\epsilon_I = (0.00417 - N_f 0.00025)\ GeV^4. \tag{14}$$

Thus one can conclude in that the standard value of the bag constant is determined by the instanton component of the nonperturbative QCD vacuum only. Note that for $N_f = 3$ it

coincides with the estimate of the QCD sum rules approach on account of the phenomenological value of the gluon condensate [20]. From (14) it also follows that the difference between values of the standard bag constant for different values of N_f is rather small but not negligible. But the main problem with the value of the bag constant, as given by expression (14), is, of course, its wrong dependence on N_f. It decreases with increasing N_f that defies a physical interpretation of the bag constant as the energy per unit volume. That is why the value of the bag constant at the expense of the instanton contributions only is at least not complete.

Instantons are classical solutions to the dynamical equations of motion of nonabelian gluon fields which contribute to the nonperturbative vacuum energy density. Therefore they are unable to explain confinement phenomenon, which, no doubt, is a quantum nonperturbative effect. The QCD vacuum has a much more remarkable (richer) topological structure than instantons alone can provide. The dynamical mechanisms of such nonperturbative effects as quark confinement and DCSB are closely related to the complicated topological large scale structure of the QCD true vacuum. Assuming that the low-frequency modes of the Yang-Mills fields can be enhanced due to the possible nonperturbative IR divergences in the true QCD vacuum [21], we have recently proposed the zero modes enhancement (ZME) model of the true QCD vacuum [2]. This is based on the solution to the Schwinger-Dyson (SD) equation for the quark propagator in the infrared domain. We have shown that this model reveals several desirable and promising features. A single quark (heavy or light) is always off mass-shell, i.e. the quark propagator has no poles. It also implies DCSB at the fundamental quark level, i.e. a chiral symmetry preserving solution is forbidden and a chiral symmetry violating solution is required. We have calculated contributions to the vacuum energy density at log-loop level [2], coming from the confining quarks with dynamically generated masses, ϵ_q, and of the nonperturbative gluons, ϵ_g, due to the enhancement of zero modes. These contributions are almost the same and the sum is $\epsilon = \epsilon_q + \epsilon_g = -0.0015\ GeV^4 - 0.0016\ GeV^4 = -0.0031\ GeV^4$. Thus the value of the bag constant, as given by the ZME model, is

$$\tilde{B}_{ZME} = -(\epsilon_g + N_f \epsilon_q) = (0.0016 + N_f 0.0015)\ GeV^4, \tag{15}$$

where we introduced the dependence on N_f since ϵ_q itself gives a single confining quark contribution to the vacuum energy density. However, neither contributions (15) nor (14) are complete. In the above mentioned papers [2], it was already explained in detail why the instanton-type fluctuations are needed for the ZME model. It has been proposed there to add $\tilde{B}_I$, given by Eq. (14), to ZME model value (15) in order to get a more realistic value of the bag constant. Thus one obtains

$$\tilde{B} = \tilde{B}_I + \tilde{B}_{ZME} = -\epsilon_t = -(\epsilon_I + \epsilon_g + N_f \epsilon_q) = (0.00577 + N_f 0.00125)\ GeV^4. \tag{16}$$

Numerically our values are as follows

$$\begin{aligned}
\tilde{B}(N_f = 0) &\simeq 0.006\ GeV^4 \simeq (278\ MeV)^4 \simeq 0.78\ GeV/fm^3, \\
\tilde{B}(N_f = 1) &\simeq 0.007\ GeV^4 \simeq (290\ MeV)^4 \simeq 0.91\ GeV/fm^3, \\
\tilde{B}(N_f = 2) &\simeq 0.008\ GeV^4 \simeq (300\ MeV)^4 \simeq 1.04\ GeV/fm^3.
\end{aligned} \tag{17}$$

We will use these values of the bag constant. Evidently, our value (16) overestimate the MIT bag model value of the bag constant [9] at least by one order of magnitude.

All values of the bag constant below the so-called standard value (14) as well as the standard value itself should be ruled out since it does not account for all components of the true QCD vacuum, as we pointed out above (see also Refs. [2]). There exist already phenomenological estimates [22] as well as lattice calculations [23] preffering a bigger-than-standard value of the bag constant. The above described components produce the main (leading) contribution to the vacuum energy density and consequently to the bag constant. The next-to-leading contributions (as given by the effective potential for composite operators at two-loop level [24]) are h^2-order, where h is the Plank constant. Thus they are suppressed at least by one order of magnitude in comparison with our values (16). It has been noticed in Ref. [1] that nobody knows yet how big the bag constant might be, but generally it is thought that it is about 1 GeV/fm^3. The proposed value (17) for the most realistic case of

the two thermal quark species ($N_f = 2$), which is more or less realized in heavy ion collisions at high energies, is in agreement with this expectation.

V. Phase diagram (T_c, μ_c) for the physically relevant case $N_f = 2$, as determined from the critical curve (11), is shown in Fig. 1. For end points of this curve $(T_c, 0)$ and $(0, \mu_c)$ our data are

$$
\begin{aligned}
T_c(N_f = 0) &= 241.5\ MeV, \\
T_c(N_f = 1) &= 186.8\ MeV, \\
T_c(N_f = 2) &= 160.6\ MeV,
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
\mu_c(N_f = 1) &= 1833.8\ MeV, \\
\mu_c(N_f = 2) &= 1441.4\ MeV,
\end{aligned}
\tag{19}
$$

respectively. Thus our value of T_c for the physically relevant case of the two quark species $N_f = 2$ at $\mu_c = 0$ is in fair agreement with its best numerical estimate which comes from the observed spectrum of low-p_T secondaries in high-energy hadron-hadron collisions, according to the analysis of Hagedorn (see Ref. [25] and references therein).

Let us compare our results for T_c with the finite temperature lattice QCD simulations with two light staggered (Kogut-Susskind) quarks represented in Ref. [26]. This can be done by means of our data shown in Eq. (18) at $\mu_c = 0$ only since, to our best knowledge, until now there are no realistic lattice data available for nonzero chemical potentials. The lattice result is $T_c(N_f = 2) = 155(9)\ MeV$ with a systematic uncertanty of about 15%, so the agreement with our value (18) is rather good. From our data (18) it also follows that the critical temperature T_c for the pure $SU(3)$ gauge theory ($N_f = 0$) is much higher than for the full QCD with two flavors ($N_f = 2$). This is in agreement with lattice calculations [26, 27], as well as with arguments of a simple percolation model [27]. A rather small discrepancy between our value of the critical temperature for the pure $SU(3)$ gauge theory displayed in

Eqs. (18) for $N_f = 0$ and recent lattice calculation $T_{latt}(N_f = 0) \simeq 260\ MeV$ [28] can be explained as follows. From the constraint (11) at $\mu_c = 0$ one obtains

$$T_c = \Big(\frac{3}{(N_f+1)f_{SB}(N_f)}\Big)^{1/4} \tilde{B}^{1/4}. \tag{20}$$

In lattice approach T_c is usually calculated from the string tension in accordance with $T_c/\sqrt{\sigma} = 0.629(3)$ [29]. This ratio depends on N_f, i. e. it is smaller for $N_f = 2$ than for $N_f = 0$, while the string tension itself does not depend on N_f [27]. Using its standard value $\sqrt{\sigma} = 420\ MeV$, one obtains the above mentioned estimate. In a similar way, let us evaluate the coefficient in our expression (20) at $N_f = 0$, while for the bag constant let us abstract from its dependence on N_f and use its value for the physically relevant (at nonzero temperature) case of the two light quark species, i. e. $\tilde{B}^{1/4} = 300\ MeV$ (see Eqs. (17)). One immediately obtains $T_c = 260.6\ MeV$ in fair agreement with the above mentioned recent lattice calculations [28, 29]. However, the numerical results of quenched (pure gauge theory for staggered quarks) finite-temperature lattice QCD should be (perhaps slightly) reconsidered in the light of the "hard chiral logarithms" problem (see recent review [30] and references therein).

VI. One of the attractive features of our approach is that the thermodynamic quantities which are defined as derivatives of the thermodynamic potential $\Omega = -P$ (extensive densities, such as the entropy density, specific heat, etc) are uniquely determined at the phase boundary. However, let us begin with introducing the metric which shows the existence of nontrivial fluctuations in QGP [31, 32]. This is done in two steps. First, the specific heat matrix is defined as follows

$$\Delta = \begin{vmatrix} \partial^2 P/\partial T^2 & \partial^2 P/\partial T\partial\mu_f \\ \partial^2 P/\partial T\partial\mu_f & \partial^2 P/\partial\mu_f^2 \end{vmatrix}$$

The corresponding entropic potential P/T possesses a statistical meaning [31, 32]. The matrix of its second derivatives $g_{ik} = \partial^2(P/T)/\partial x^i\partial x^k$ with respect to the canonical coordinates $x^i \equiv (1/T, -\mu_i/T)$ defines the avarage scales of the fluctuations by $< g_{ik}\delta x^i\delta x^k >= 1$. There is an intimate connection between g_{ik} and the specific heat determinant Δ_{ik} which shows

that if the former is positive definite then the latter is also positive definite and the other way around [31, 32]. Then the fluctuations remain finite. For the noninteracting medium the specific heat determinant is always positive definite. Using Eqs. (1-2), one obtains $\Delta(T,\mu_f) = 4f_{SB}T^4 + \frac{3}{\pi^2}N_f^2\mu_f^4 + (6.4+1.2N_f)N_f T^2\mu_f^2$. At the transition phase it finally becomes (on account of Eq. (11)) $\Delta(T_c,\mu_c) = 4N_f\left[\frac{3}{N_f+1}\tilde{B} + (0.177-0.133N_f)T_c^2\mu_c^2\right]$. For $N_f = 0, 1$ it is automatically positive definite and, using our numerical results for $N_f = 2$, it is also positive. Thus there are no nontrivial fluctuations in the noninteracting QGP in the vicinity of the phase transition as it should be indeed.

The entropy density is defined as $s = \left(\frac{\partial P}{\partial T}\right)_{\mu_f}$, so, on account of Eqs. (1-2), it is, $s = \frac{4}{3}f_{SB}T^3 + N_f\mu_f^2 T$. At the phase boundary it finally becomes

$$s(T_c,\mu_c) = \frac{4}{3}f_{SB}T_c^3 + \frac{N_f}{9}\mu_c^2 T_c. \tag{21}$$

Its behaviour along the critical curve (11) is shown in Figs. 2.

The specific heat is defined as follows $c = T\left(\frac{\partial^2 P}{\partial T^2}\right)_{\mu_f}$, and again using Eqs. (1-2), one obtains $c = 4f_{SB}T^3 + N_f\mu_f^2 T$. At the phase transition it finally becomes

$$c(T_c,\mu_c) = 4f_{SB}T_c^3 + \frac{N_f}{9}\mu_c^2 T_c. \tag{22}$$

Its behaviour along the critical curve (11) is shown in Figs. 3.

The net quark number density is $n_f = \left(\frac{\partial P}{\partial \mu_f}\right)_T$. Again using Eqs. (1-2), one obtains $n_f = N_f\mu_f(T^2 + \frac{1}{\pi^2}\mu_f^2)$. Since the baryon (B) number of a quark is 1/3, the net baryon number density $n_B = \frac{1}{3}n_f$ at the phase transition finally becomes

$$n_B(T_c,\mu_c) = \frac{N_f\mu_c}{9}(T_c^2 + \frac{1}{9\pi^2}\mu_c^2). \tag{23}$$

Its behaviour along the critical curve (11) is shown in Figs. 4.

One of the important observables measured in heavy ion collisions is the specific entropy per baryon, s/n_B (see, Refs. [10, 33] and references therein). Its behaviour across and along the phase transition may shed light on the strangeness production as a signal of QGP formation in heavy ion collisions. At the phase transition it is determined as follows

$$\frac{s}{n_B}(T_c,\mu_c) = \frac{s(T_c,\mu_c)}{n_B(T_c,\mu_c)}, \tag{24}$$

where $s(T_c,\mu_c)$ and $n_B(T_c,\mu_c)$ are given by Eqs. (21) and (23), respectively. Its numerical values can be easily obtained from curves in Figs. 5. In Fig. 1, the path of constant $s/n_B \simeq 50$, which is expected from the QGP fireball, is additionally shown. From Fig. 1 it also follows that for the QGP fireball temperature $T \simeq 215 \pm 10\ MeV$ the baryochemical potential is $\mu_B \simeq 340 \pm 20\ MeV$ as it should be indeed [33].

Let us now compute the numerical values of the entropy density s and specific heat c at $\mu_c = 0$ since the dependence on temperature is more important than the dependence on chemical potential. From Eq. (21) and constraint equation (11) one obtains

$$s_c = \frac{4}{3} f_{SB}^{1/4} \Big(\frac{3}{N_f + 1} \tilde{B}\Big)^{3/4}. \tag{25}$$

Numerically this gives

$$\begin{aligned} s_c(N_f = 0) &= 0.0992\ GeV^3, \\ s_c(N_f = 1) &= 0.0751\ GeV^3, \\ s_c(N_f = 2) &= 0.0666\ GeV^3. \end{aligned} \tag{26}$$

The numerical values for the specific heat at the phase boundary c_c may be obtained from Eq. (26) in accordance with the relation $c_c = 3s_c$, which comes from Eqs. (21) and (22).

VII. Here let us develop a general method of calculating the parametric function $b_h(T_c,\mu_c)$ which remains still unknown in Eqs. (12-13). This makes it also possible to calculate the bulk thermodynamic quantities such as the critical energy density, ϵ_c, the latent heat, ϵ_{LT}, etc, in the vicinity of the transition point $T = T_c$.

Since the parametric function $b_h(T_c,\mu_c)$ describes details of the HG phase structure, in order to determine it, one, evidently, needs to choose the concrete equation of state of the HG phase. In this part of our work in what follows, $P_h(T_c)$ will denote the concrete equation of state of the HG phase at $T = T_c$ for point-like particles. In order to take into account extended volume corrections [18] it should be divided by factor $[1 + V_0 n_h(T,\mu)]$,

where $V_0 = (4\pi R_0^3/3)$ with $R_0 \sim 0.8\ fm$, is the volume of nucleon, and $n_h(T, \mu)$ is the baryon number density for the point-like particles, i. e. it should be calculated from P_h as follows

$$n_h(T, \mu) = \Big(\frac{\partial P_h}{\partial \mu}\Big)_T. \tag{27}$$

The more realistic hadron pressure thus becomes

$$P_H = \frac{P_h}{[1 + V_0 n_h(T, \mu)]}. \tag{28}$$

Solving now Eq. (12) against b_h with substitution $P_h \to P_H$, one obtains

$$b_h \equiv b_h(T_c, \mu_c) = \frac{(N_f + 1)^2 P_H(T_c, \mu_c)}{\tilde{B} - (N_f + 1) P_H(T_c, \mu_c)}. \tag{29}$$

So this relation allows one to calculate $b_h(T_c, \mu_c)$ on account of the chosen equation for the HG phase, P_H, by taking the values of T_c and μ_c from the general constraint (11). Substituting, such calculated value of $b_h(T_c, \mu_c)$, back to Eqs. (12-13), one obtains the numerical values of $P_q(T_c)$ and the critical energy density ϵ_c in our model. In order to calculate the latent heat, it is necessary first to calculate the energy density of the HG phase as follows

$$\epsilon_h = T\Big(\frac{\partial P_h}{\partial T}\Big)_\mu + \mu\Big(\frac{\partial P_h}{\partial \mu}\Big)_T - P_h. \tag{30}$$

Similar to P_H, the more realistic energy density becomes

$$\epsilon_H = \frac{\epsilon_h}{[1 + V_0 n_h(T, \mu)]}. \tag{31}$$

Then the more realistic value of the latent heat is $\epsilon_{LH} = \epsilon_c - \epsilon_H$, where ϵ_c is given by Eq. (13). This is a general scheme how to calculate $\epsilon_{LH}, \epsilon_c$, etc within the extended bag model. Let us note that our Ansatz (5-7) automatically incorporates extended volume corrections.

VIII. In this part let us explicitly show how the above formulated general method works by approximating the hadronic phase by the ideal (noninteracting) gas of massless mesons. The state equation in this case is [11, 12, 34]

$$P_h = \frac{1}{3}\epsilon_h = g_h \frac{\pi^2}{90} T^4, \tag{32}$$

where g_h is the number of hadronic degrees of freedom. Evidently, in this case there are no extended volume corrections, i. e. $P_h = P_H$ and $\epsilon_h = \epsilon_H$. From Eq. (29), on account of Eq. (11) at $\mu_c = 0$ and Eq. (32) at $T = T_c$, one finally obtains

$$b_h = \frac{(N_f + 1) g_h \pi^2}{30 f_{SB} - g_h \pi^2}, \tag{33}$$

i. e. in this simple case, the parametric function becomes constant, not depending on T_c. The critical energy density (13) then becomes

$$\epsilon_c = \frac{120 f_{SB} - g_h \pi^2}{30 (N_f + 1) f_{SB}} \tilde{B}. \tag{34}$$

The meson gas energy density at $T = T_c$, on account of Eqs. (32) and (11), is

$$\epsilon_h = 3P_h = \frac{g_h \pi^2}{10 (N_f + 1) f_{SB}} \tilde{B}, \tag{35}$$

and the LH becomes

$$\epsilon_{LH} = \epsilon_c - \epsilon_h = \frac{30 f_{SB} - g_h \pi^2}{7.5 (N_f + 1) f_{SB}} \tilde{B}. \tag{36}$$

Thus these expressions allow one to calculate the bulk thermodynamic quantities in the vicinity of the phase transition in terms of the fundamental quantity, the bag constant $\tilde{B}$. The numerical results in the massless pion gas limit are ($g_h = 3$, $N_f = 2$): $\epsilon_c = 1.358 \ GeV/fm^3$, $\epsilon_h = 0.084 \ GeV/fm^3$, $\epsilon_{LH} = 1.274 \ GeV/fm^3$, in fair agreement with Ref. [34]. This can be explained by the fact that the authors of the above mentioned paper *a priori* use the value of T_c which coincides with the value calculated with our method and shown in Eq. (18) for $N_f = 2$. By construction, the phase transition is of first order, i. e. ϵ_{LH} is discontinuous (finite).

Let us now calculate s_h and c_h in this model in order to estimate the jumps in these quantities between the HG and QGP phases. The entropy density in the hadronic phase is defined as $s_h = \partial P_h / \partial T$, where P_h is given by Eq. (32). On account of the constraint equation (11) at $\mu_c = 0$, one finally obtains

$$s_h(T_c) = \frac{g_h \pi^2}{22.5} \Big(\frac{3 \tilde{B}}{(N_f + 1) f_{SB}} \Big)^{3/4}. \tag{37}$$

Its numerical value at $g_h = 3, N_f = 2$ is: $s_h = 0.0054\, GeV^3$. The specific heat in the hadronic phase is defined as $c_h = T\Big(\partial^2 P_h/\partial T^2\Big)$. Again on account of the constraint equation (11) at $\mu_c = 0$, one obtains $c_h(T_c) = 3s_h(T_c)$, where $s_h(T_c)$ is given by Eq. (37). So at $g_h = 3, N_f = 2$ numerically it is $s_h = 0.0162\ GeV^3$. In the HG phase these numbers thus are by one order of magnitude less than the corresponding numbers in the QGP phase (see Eqs. (26)).

It is instructive to compare the numerical value of T_c which follows from the standard bag model with that of the extended bag model given in Eqs. (18) for $N_f = 2$. In the standard bag model the pressure, as given by Eq. (1) at $\mu_f = 0$, should be directly equated to P_h, Eq. (32), at $T = T_c$. This gives the constraint equation as follows

$$T_c = \Big(\frac{90}{30 f_{SB} - g_h \pi^2}\Big)^{1/4} B^{1/4}. \tag{38}$$

Assigning now our value of the bag constant (17) to B, for the massless pion gas limit ($g_h = 3,\ N_f = 2$) one obtains $T_c \simeq 216\ MeV$. Of course, this value substantially contradicts our value (18) as well as the above presented lattice result (part V).

IX. Let us consider now a more sophisticated model of the HG phase which consists of massless pion gas and nucleons, antinucleons with masses m. The thermodynamic potential in this case is [10]

$$P_h = \frac{\pi^2}{30} T^4 + \frac{4m^2}{\pi^2} T^2 \sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{l^2} K_2(l\frac{m}{T}) \cosh(l\frac{\mu}{T}), \tag{39}$$

where K_2 is the modified Bessel function of the second kind and the first term describes the massless pion gas limit. This is the expression for the point-like particles and P_H on account of the extended volume corrections should be obtained from Eq. (28). Using now definition (27), one has

$$n_h(T, \mu) = \frac{4m^2}{\pi^2} T \sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{l} K_2(l\frac{m}{T}) \sinh(l\frac{\mu}{T}). \tag{40}$$

The energy density ϵ_h, on account of Eqs. (30) and (39), becomes

$$\epsilon_h = \frac{\pi^2}{10} T^4 + \frac{4m^2}{\pi^2} T^2 \sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{l^2} [K_2(l\frac{m}{T}) - l\frac{m}{T} K_2^{'}(l\frac{m}{T})] \cosh(l\frac{\mu}{T}), \tag{41}$$

where prime denotes the differentiation with respect to the argumentum. Similarly to the previous case, ϵ_H is to be obtained from Eq. (31) on account of this expression. These relations (39-41) are convinient enough to describe the hot HG and low density matter ($\mu \to 0$). For cold ($T = 0$) and baryon dense matter, it becomes a fully degenerate Fermi gas at zero temperature. So the finite temperature corrections can be found by using a power expansion around $T = 0$ [10].

Expression (29) for b_h, in general, becomes

$$b_h \equiv b_h(T_c, \mu_c) = \frac{(N_f + 1)^2 P_h(T_c, \mu_c)}{\tilde{B}[1 + V_0 n_h(T_c, \mu_c)] - (N_f + 1) P_h(T_c, \mu_c)}, \tag{42}$$

where $P_h(T_c, \mu_c)$ and $n_h(T_c, \mu_c)$ are given by Eqs. (39) and (40), respectively with the substitutions $T \to T_c$ and $\mu \to \mu_c$. Thus, in comparison with the previous case (part VII), now the parametric function is not simply a constant but it depends crucially on T_c and μ_c which should be taken from the general constraint (11) in order to numerically calculate $b_h(T_c, \mu_c)$ from this expression.

Let us calculate, in this model, ϵ_c and ϵ_{LH} at $\mu_c = 0$ which allows us to compare numerical results with those of the previous part as well as with lattice results (see below). In this limit extended volume corrections disappear and one can use classical statistics for nucleons, i. e. only the first term in the series expansion (39) [10]. On account of the constraint equation (11) at $\mu_c = 0$, then from Eq. (42), one finally obtains ($N_f = 2$ everywhere below)

$$b_h(T_c) = \frac{3[\frac{\pi^2}{10} + \frac{12}{\pi^2} m_c^2 K_2(m_c)]}{f_{SB}(2) - [\frac{\pi^2}{10} + \frac{12}{\pi^2} m_c^2 K_2(m_c)]}, \tag{43}$$

where $m_c = m/T_c$ with $m = (m_P + m_N)/2$ and $T_c = 160.6\ MeV$ (see Eqs. (18)). In a similar way, from Eq. (41) ϵ_h becomes

$$\epsilon_h(T_c) = \frac{1}{f_{SB}(2)} \Big(\frac{\pi^2}{10} + \frac{4m_c^2}{\pi^2} [K_2(m_c) - m_c K_2^{'}(m_c)] \Big) \tilde{B}. \tag{44}$$

Numerically one get $\epsilon_h = 0.1\ GeV/fm^3$. The critical energy density is to be calculated from Eq. (13) on account of the numerical value of b_h, obtained from expression (43). This is $\epsilon_c = 1.35\ GeV/fm^3$, so the numerical value of the latent heat is $\epsilon_{LH} = 1.25\ GeV/fm^3$.

The value of the bag constant as given in Eqs. (17) for $N_f = 2$ was usedi n all numerical calculations above. These values differ only slightly from those obtained in the previous part. Despite the simple models considered above, these numbers look quite reasonable and they may be compared with lattice result especially for ϵ_c, discussed for example, in Ref. [35]. The latent heat in this model also remains discontinuous, thereby confirming the first order nature of the phase transition. The entropy density s_h as well as specific heat c_h, in this model also differ very slightly from those calculated in the previous model.

An extension of our model to the case of running coupling constant in order to treat interacting QGP (which has been already rather tentatively discussed in Ref. [36]) along with an extrapolation outside the phase boundary in order to constract equilibrium phase transition [10] are subjects of the subsequent papers.

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FIGURES

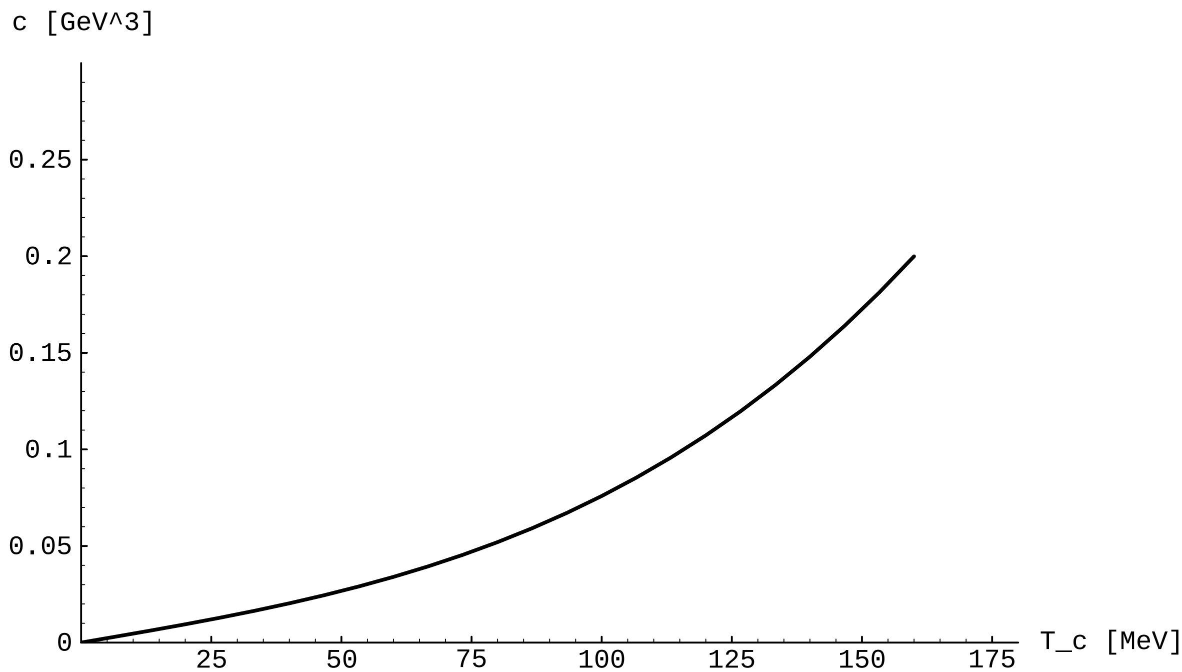
FIG. 1. The phase diagram in the plane (T_c, μ_c) measured in inits of MeV. Here and in all diagrams below the curves are only shown for the physically relevant case of the two light quarks $N_f = 2$ (solid lines). Also shown the path of constant specific entropy per baryon $s/n_B \simeq 50$ (dashed-dotted line).

FIG. 2. The entropy density as a function of T_c (upper figure) and μ_c (lower figure).

FIG. 3. The specific heat as a function of T_c (upper figure) and μ_c (lower figure).

FIG. 4. The baryon number density as a function of T_c (upper figure) and μ_c (lower figure).

FIG. 5. The specific entropy per baryon s/n_B as a function of T_c (upper figure) and μ_c (lower figure).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150
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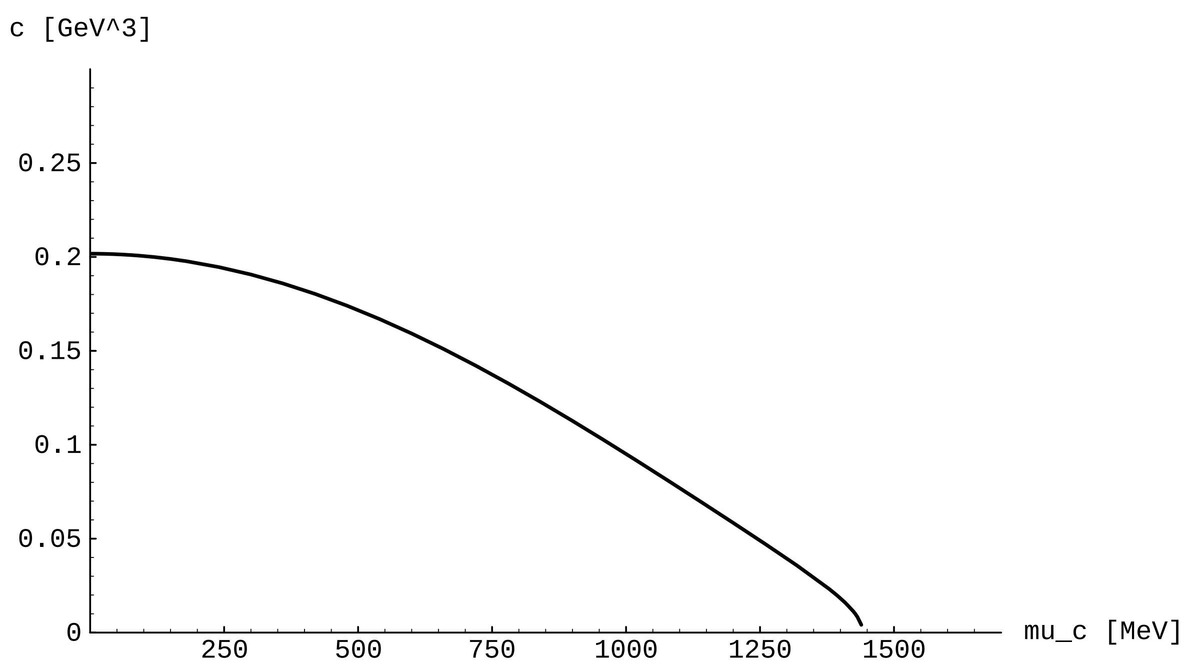
T_c [MeV]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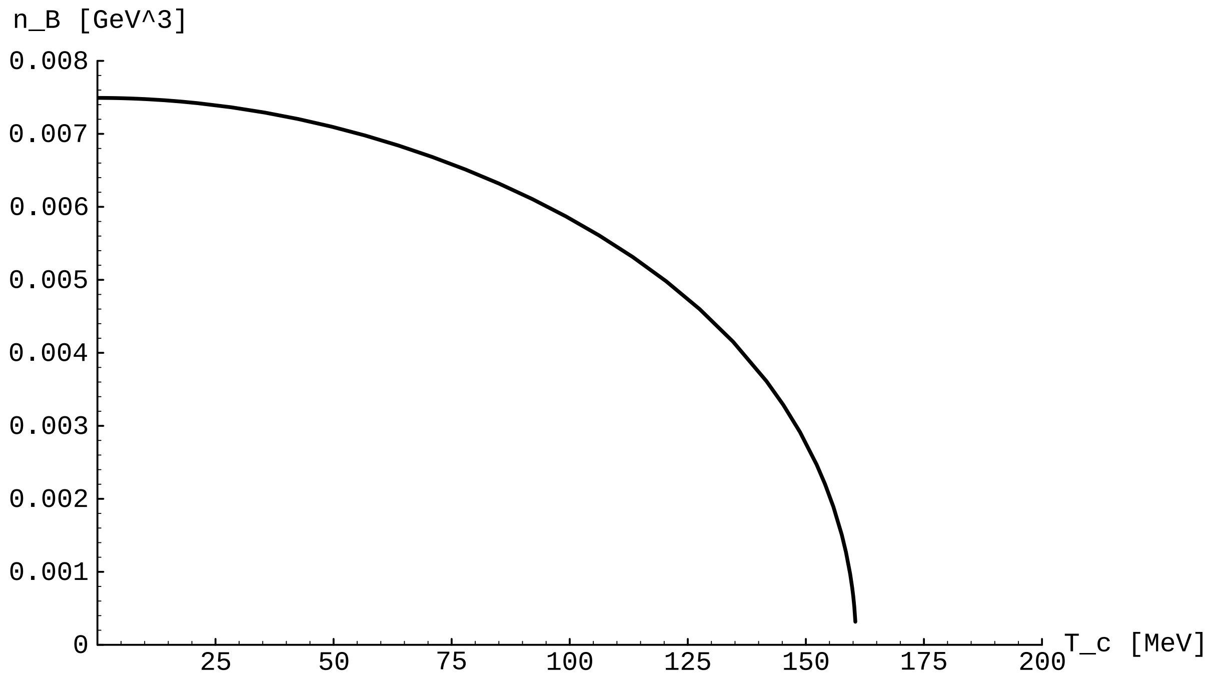
mu_c [MeV]



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0.007
0.006
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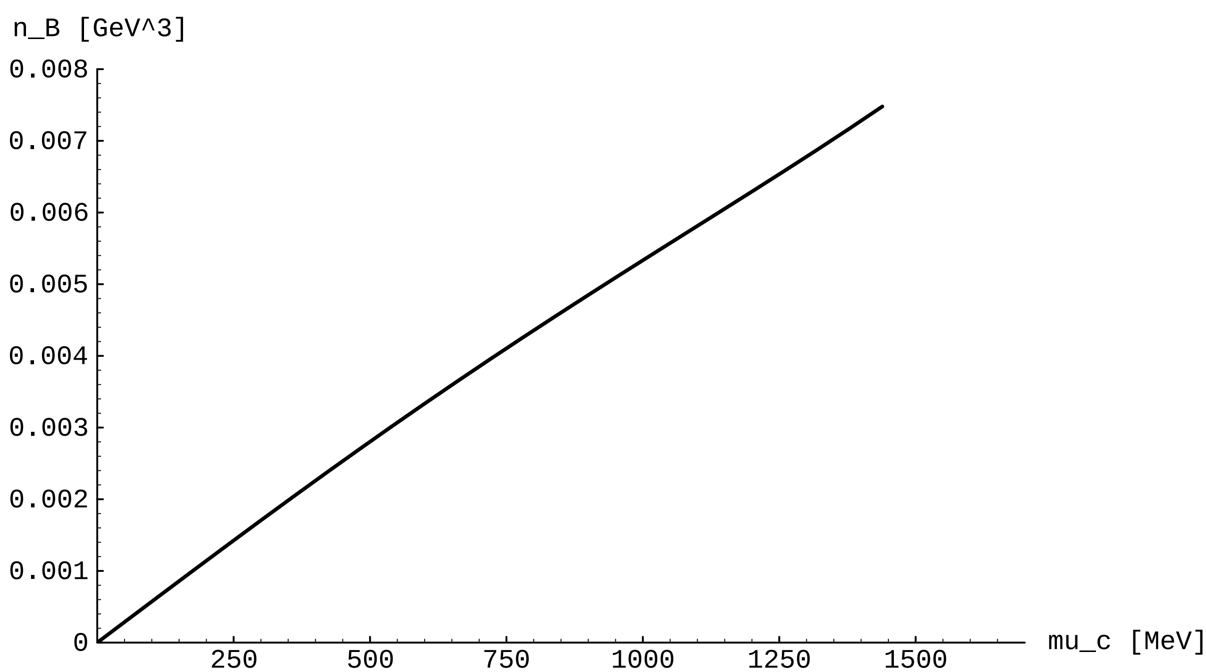
75
100
125
150
175
200
T_c [MeV]



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0.007
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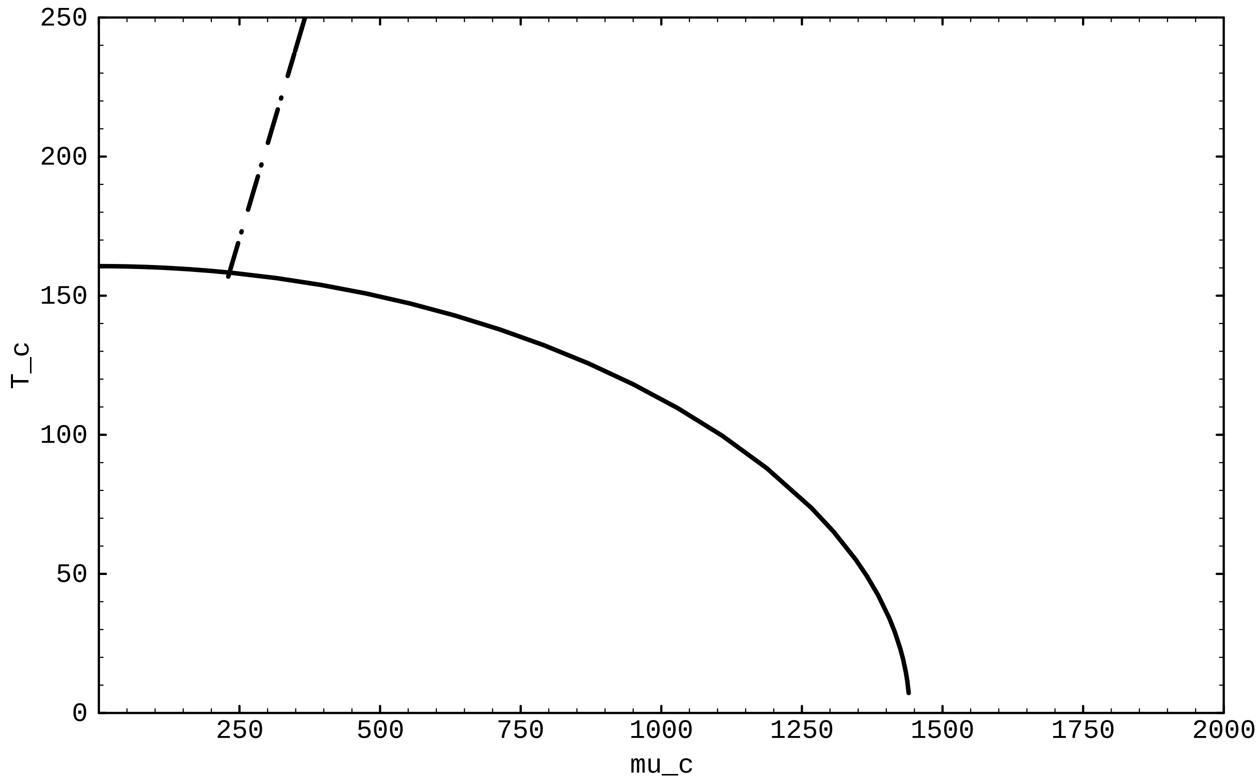
mu_c [MeV]



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T_c
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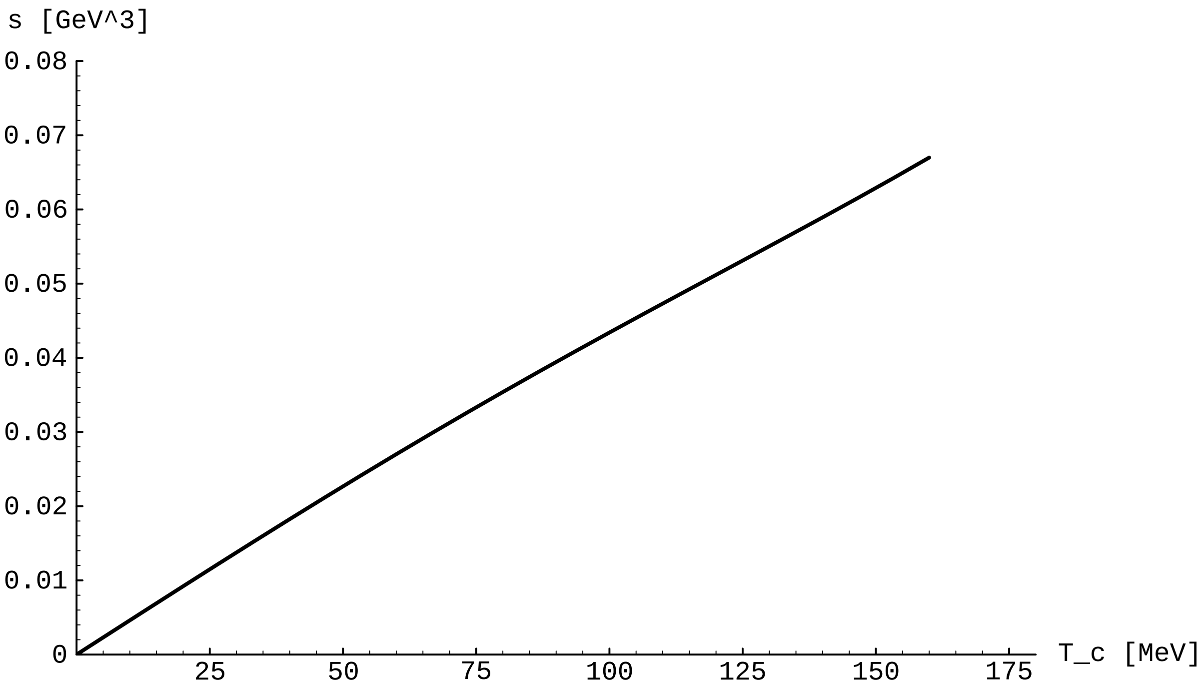
1750
2000
mu_c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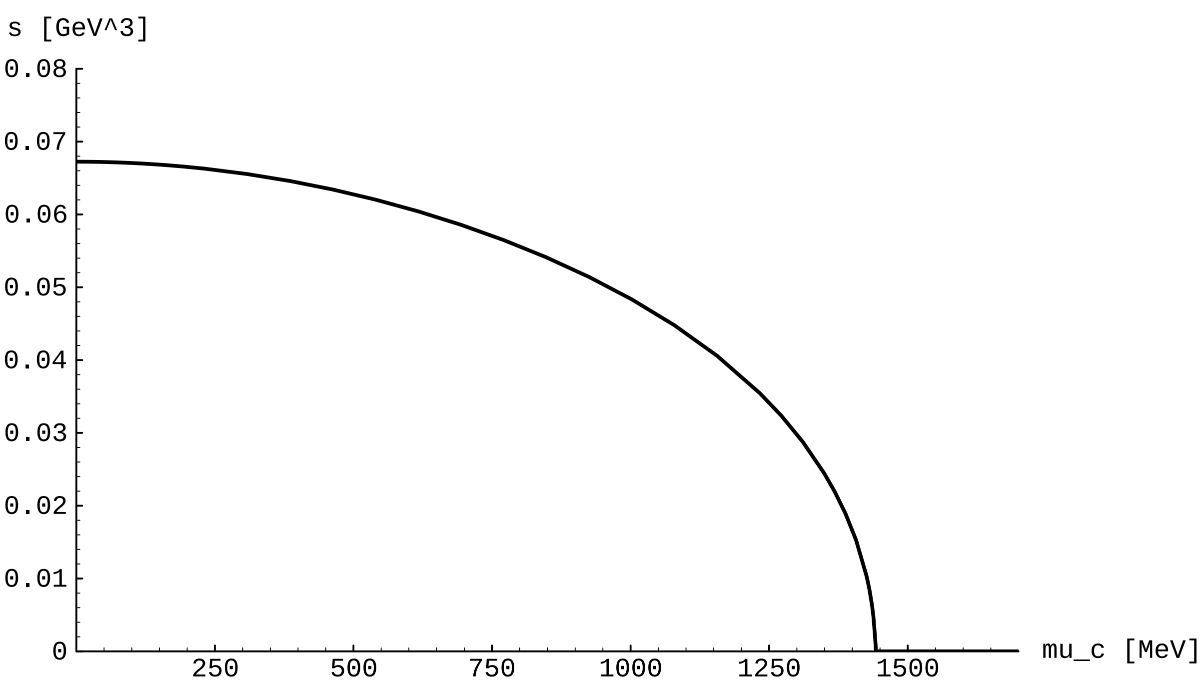
T_c [MeV]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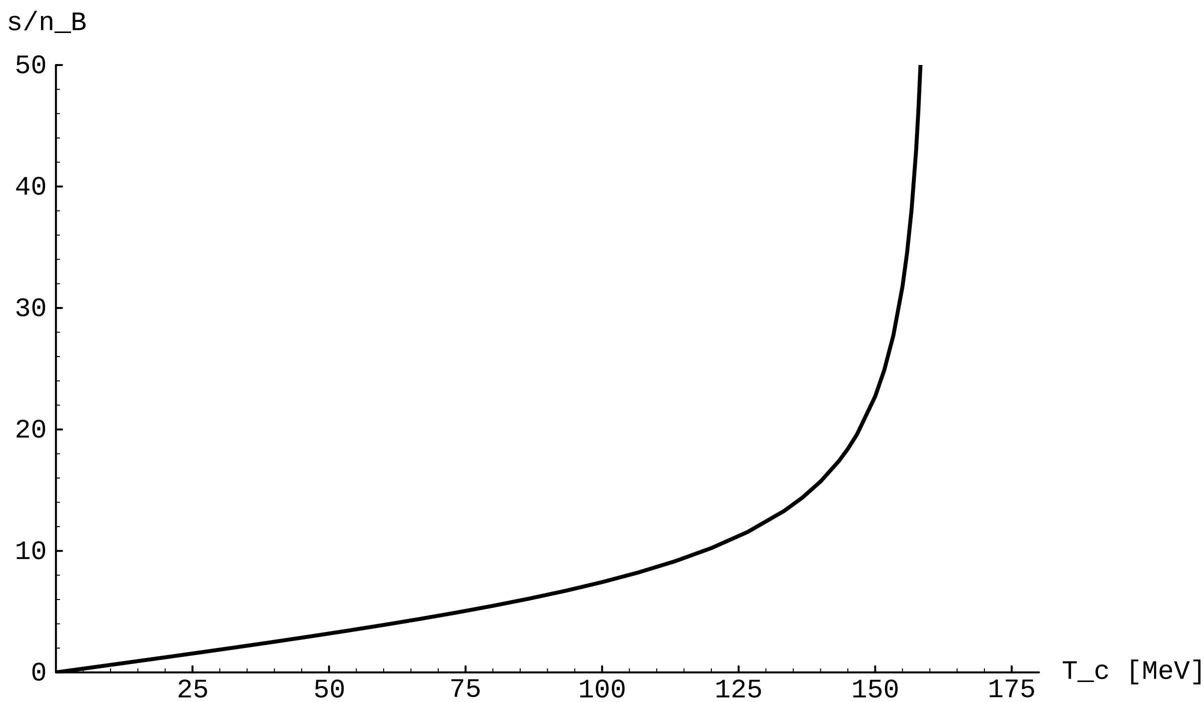
mu_c [MeV]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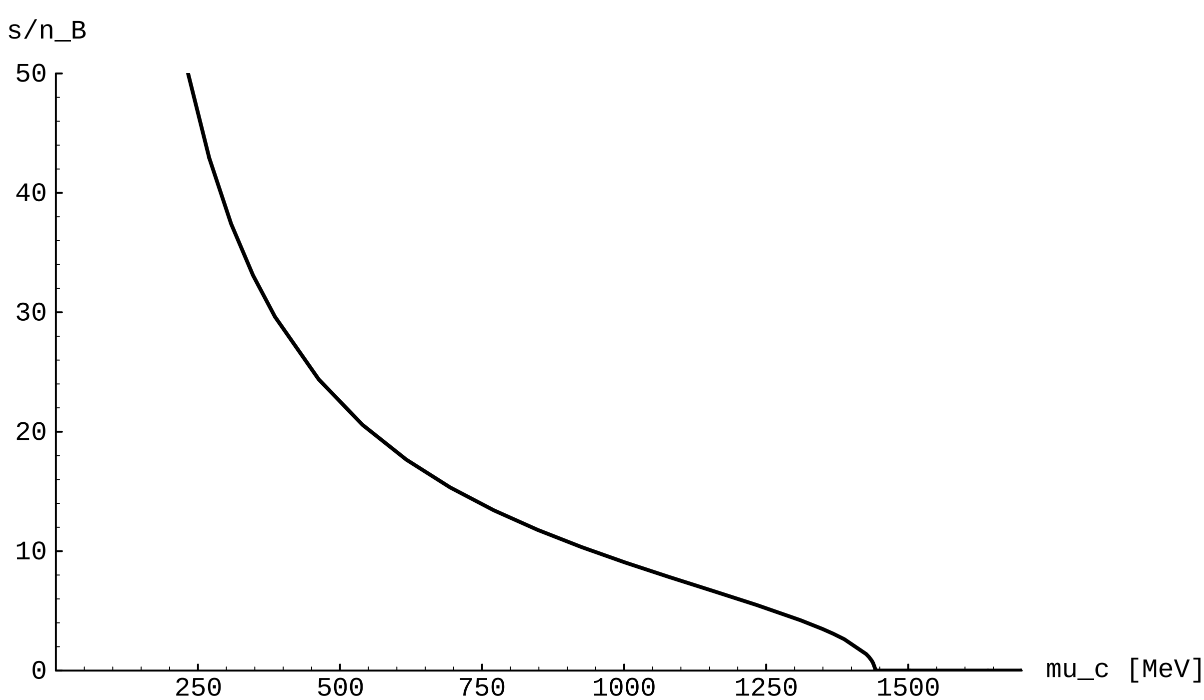
T_c [MeV]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mu_c [MeV]